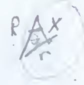
EUF 2012 - 1sem 27/03/2016

Q1

a)

Inercia esfera oca



$I = \int r^2 dm$

$\sin\theta = \frac{r}{R}$

$dm = \rho \cdot dA = \rho \cdot 2\pi r \cdot R d\theta$

$dm = 2\pi \sin\theta R^2 d\theta \Rightarrow I = \int R^2 \sin^2\theta \sin\theta 2\pi R^2 d\theta$

$L = r \times p = I\omega$

$I = 2\pi R^4 \rho \int_0^{\pi} \sin^3\theta \, d\theta = 2\pi R^4 \rho \cdot \frac{4}{3} = \boxed{\frac{2}{3} R^2 M}$

$I_T = \left(\frac{2}{3} MR^2 + Md_1^2\right) 2$ ⇝ duas esferas

$d_1 = \frac{d_0}{2}$ 4R

$2 \cdot I_T = \frac{2}{3} MR^2 + 16 MR^2 = \boxed{\frac{100 MR^2}{3}} \Rightarrow \boxed{L = \frac{100 MR^2}{3} \cdot \omega}$

$\omega = \frac{2\pi}{T_0}$

b) $L_i = L_f$ $d = \frac{R}{2}$

$I_T' = 2 \cdot \left(\frac{2}{3} MR^2 + \frac{MR^2}{4}\right) = \frac{22 MR^2}{12} = \frac{11 MR^2}{6}$

$\frac{100 MR^2 \omega_i}{3} = I_T' \omega_f \Rightarrow \omega_f = \frac{100 MR^2 \omega_i}{3} \cdot \frac{6}{11 MR^2} \Rightarrow \boxed{\omega_f = \frac{300 \omega_i}{11}}$

c) $K = \frac{1}{2} I \omega^2$

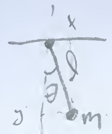
$K_i = \frac{1}{2} \cdot \left[\frac{100 MR^2}{3}\right] \cdot \omega_i^2$

$\Rightarrow \Delta K = K_f - K_i = \frac{\omega_i^2 MR^2}{6}\left[150 - 100\right] = \boxed{\frac{25 \omega_i^2 MR^2}{3}}$

$K_f = \frac{1}{2} \cdot \frac{11 MR^2}{6} \cdot \omega_f^2$

d) $W = \Delta K$

Q2.

a) 

$x = \sin\theta \cdot l \rightarrow \dot{x} = l \cdot \dot{\theta}\cos\theta$

$y = -\cos\theta \cdot l \rightarrow \dot{y} = +l\dot{\theta}\sin\theta$

$$T = \frac{1}{2}m(\dot{x}^2+\dot{y}^2) = \frac{1}{2}ml^2\dot{\theta}^2$$

$U = mgh = -mgl\cos\theta \Rightarrow L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl\cos\theta$

$ml^2\ddot{\theta} + mgl\sin\theta = 0$ or: $x = l\theta$, $a = l\ddot{\theta}$

$$\boxed{\ddot{\theta} + \frac{g}{l}\theta = 0}$$

$F = ma$

$ml\ddot{\theta} = +mg\sin\theta \Rightarrow \boxed{\ddot{\theta} - \frac{g}{l}\theta = 0}$

$$\boxed{\omega^2 = \frac{g}{l}}$$

b) $r^2 = \frac{g}{l} \Rightarrow r = \pm i\sqrt{\frac{g}{l}}$

$\theta(t) = A\cos\left(\sqrt{\frac{g}{l}}t\right) + B\sin\left(\sqrt{\frac{g}{l}}t\right) \Rightarrow$ $\theta(0) = 0$, $\frac{d\theta(0)}{dt} = \Omega$

$\dot{\theta}(t) = B\sqrt{\frac{g}{l}}\cos\left(\sqrt{\frac{g}{l}}t\right) \Rightarrow \boxed{B = \Omega\sqrt{\frac{l}{g}}}$

$$\boxed{\theta(t) = \frac{\Omega}{\omega}\cos(\omega t)}$$

c) 

$-mg\sin\theta + 2m\sqrt{gl}\,\dot{\theta} = F \Leftrightarrow m \cdot a = F$

$ml\ddot{\theta} = 2m\sqrt{gl}\,\dot{\theta} - mg\sin\theta$

$$\boxed{\ddot{\theta} - 2\sqrt{\frac{g}{l}}\,\dot{\theta} - \frac{g}{l}\theta = 0}$$

2

d) $r^2 - 2wir - w^2 = 0$

$\Delta = 4w^2 \cdot 4w^2 = 0 \Rightarrow \boxed{r = w} \rightarrow \theta(t) = tAe^{iwt} + Be^{iwt}$

$\theta(0) = \theta_0 \quad \dot\theta(t) = Ae^{iwt} + iwtAe^{iwt} + iwBe^{iwt}$

$\dot\theta(0) = 0$

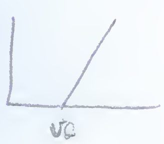
$\dot\theta(0) = 0 = A + iwB \Rightarrow A = -iwB$

$\theta(0) = \theta_0 = B \rightarrow A = -iw\theta_0$

$\Rightarrow \boxed{\theta(t) = -iw\theta_0 \cdot t e^{iwt} + \theta_0 e^{iwt}}$

Q3. 1 ms duração, $\lambda = 414$ nm, $P_0, 3P_0, 5P_0$, $P_0 = 300$ keV/s

a)



$K_{max} = eV_0 - \phi$

emissão de fotoelétrons não depende da potência, apenas da freq.

Potência aplicada $\Rightarrow P = 300 \cdot 10^3$ eV/s $\cdot 10^{-3}$ s $= 300$ eV

$E = \frac{h \cdot c}{\lambda} = \frac{4{,}14 \cdot 10^{-15} \cdot 3 \cdot 10^8}{4{,}14 \cdot 10^{-7}} = 3$ eV

Irá ocorrer emissão apenas p/ Li

b) $N_{photons} = \frac{P_{laser}}{E} = \frac{\lambda}{hc} P_{laser} = \frac{900 \text{ eV}}{3 \text{ eV}} = 300$ photons sendo emitido do laser

↳ $\frac{\text{Potência do laser}}{\text{"Potência" de cada photon}}$

Considerando $\eta = 1 \Rightarrow$ poderão ser emitidos 300 photons das placas de Lítio.

c) $Sc \to 1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^1$

wtl...

$4s^2$ $3d$

$n=4$ $n=3$

$l=0$ $l=2$

d) $L_z = m\hbar$

$m = 0, \pm 1, \pm 2, \pm l$ $\Rightarrow$ $3d^1$ $n=3$ $l=2$ $\Rightarrow$ $\boxed{L_z = -2\hbar, -\hbar, 0, \hbar, 2\hbar \quad L = \sqrt{6}\hbar}$

$L = \sqrt{l(l+1)}\hbar \Rightarrow$

Q4.

a) $\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi(x) = 0 \to k^2 = \frac{2mE}{\hbar^2}$

$\psi(0) = 0$ $\psi'(0) = 0$

$\psi(d) = 0$ $\psi'(d) = 0$

b) $\psi(x) = A\sin(kx) + \cancel{B\cos(kx)}$

$A\sin(kd) = 0 \Rightarrow \boxed{k = \frac{n\pi}{d}} \Rightarrow \boxed{\psi(x) = \sqrt{\frac{2}{d}}\sin\left(\frac{n\pi}{d}x\right)}$

$\int_0^d A^2 \sin^2(kx)\,dx = 1 \Rightarrow \frac{A^2 d}{2} = 1 \Rightarrow A = \sqrt{\frac{2}{d}}$ $E_n = \frac{n^2\pi^2\hbar^2}{d^2 2m}$

c) $\psi(x) = \sqrt{\frac{2}{d}}\sin\left(\frac{3\pi x}{d}\right)$; $\boxed{n=3} \to \boxed{E_3 = \frac{9\pi^2\hbar^2}{2md^2}} \to E_3 = \frac{hc}{\lambda} \Rightarrow \boxed{\lambda = \frac{hc}{E_3}}$

d) $P(0<x<\frac{d}{6}) = \int_0^{d/6} |\psi(x)|^2 dx = \frac{2}{d}\int_0^{d/6} \sin^2\left(\frac{3\pi x}{d}\right)dx = \frac{1}{d}\left[\frac{d}{6} - \frac{d}{6\pi}\cdot(\sin(\pi) - \sin(0))\right]$

$\boxed{= \frac{1}{6}}$

14

EVF 2012 - 1sem 27/03/2016

Q6.

$a<r<b$ $r>b$

a) $B = \frac{\mu_0 I}{2\pi r}$ b) $B = 0$

c) $B = \frac{\mu_0 I_{enc}}{2\pi r}$ $I_{enc} = \frac{4\pi r^2}{4\pi a^2} \cdot I$ $\Rightarrow \boxed{B = \frac{\mu_0 r I}{2\pi a^2}}$

$H = \frac{B}{\mu}$

d) $u_B = \frac{1}{2}\mu H^2 = \frac{1}{2}\frac{B^2}{\mu} = \frac{1}{2\mu}\left(\frac{\mu_0 I}{2\pi r}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 r^2}$

$U_B = \iiint u_B \, dV = \int_a^b \frac{\mu_0 I^2}{8\pi^2 r^2} 2\pi r l \, dr = \frac{2\pi \mu_0 l}{8\pi^2} \ln\left(\frac{b}{a}\right)$

$\boxed{\frac{U_B}{l} = \frac{\mu_0 \ln\left(\frac{b}{a}\right)}{4\pi}}$

Q7.

a) metade superior

$E = \frac{Q}{4\pi r^2 \varepsilon_0}$ $K = \frac{\varepsilon}{\varepsilon_0}$

inf.

$E = \frac{Q}{4\pi r^2 K \varepsilon_0}$

b) $V = \frac{Q}{4\pi r \varepsilon}$ $\Rightarrow \sigma_{f} 4\pi r_a^2 = Q$ $D = \frac{Q}{4\pi r^2}$ $\nabla D = \rho_f$

$\rho_f = 0$

$\sigma_f = \frac{Q}{4\pi r_a^2}$ e $-\frac{Q}{4\pi r_b^2}$

c) $\sigma_b = P \cdot \hat{n}$ $P = \varepsilon_0 \chi_e E = \frac{\varepsilon_0 \chi_e \cdot Q}{4\pi r^2 \varepsilon_0}$

$\sigma_b = \begin{cases} \frac{\chi_e Q}{4\pi r_a^2} \hat{r} \\ -\frac{\chi_e Q}{4\pi r_b^2} \hat{r} \end{cases}$

d) a densidade de carga de polarização sobre a superfície plana do dielétrico é nula porque $\hat{z} \perp \hat{r}$

e) $C = \frac{Q}{V_A - V_B} = \frac{Q}{\frac{Q}{4\pi\varepsilon r_a} - \frac{Q}{4\pi\varepsilon r_b}} = \frac{4\pi\varepsilon (r_a r_b)}{(r_b - r_a)}$

$\boxed{5}$

Q8. a) $\boxed{a\Psi_0 = 0}$

$$\sqrt{\frac{m\omega}{\hbar}}\, x\,\Psi_0 + \sqrt{\frac{\hbar}{m\omega}}\,\frac{d\Psi_0}{dx} = 0$$

$$\frac{d\Psi_0}{dx} + \frac{m\omega}{\hbar}\Psi_0 \cdot x = 0 \Rightarrow \frac{d\Psi_0}{dx} = -\frac{m\omega x}{\hbar}\Psi_0$$

$$\frac{d\Psi_0}{\Psi_0} = \frac{-m\omega x\,dx}{\hbar} \Rightarrow \ln\Psi_0 = \frac{-m\omega x^2}{2\hbar}$$

$$\boxed{\Psi_0 = A\,e^{\frac{-m\omega x^2}{2\hbar}}}$$

b) $$\int_{-\infty}^{\infty} A^2 e^{\frac{-m\omega x^2}{\hbar}}\,dx = 1$$

$\alpha = \frac{m\omega}{\hbar}$

$$A^2 \cdot \left(\frac{\pi\hbar}{m\omega}\right)^{\frac{1}{2}} = 1 \Rightarrow \boxed{A = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}}$$

c) $$\Psi_0' = -\frac{m\omega x}{\hbar} A\, e^{\frac{-m\omega x^2}{2\hbar}}$$

$$\Psi_0'' = -\frac{m\omega}{\hbar} A\, e^{\frac{-m\omega x^2}{2\hbar}} + \frac{m\omega x}{\hbar}\cdot\left(\frac{m\omega x}{\hbar}\right)\cdot A\, e^{\frac{-m\omega x^2}{2\hbar}}$$

$$-\frac{\hbar^2}{2m}\left[-\frac{m\omega}{\hbar}\cancel{A e^{-\beta x^2}} + \left(\frac{m\omega x}{\hbar}\right)^2 \cancel{A e^{-\beta x^2}}\right] + \frac{1}{2}m\omega^2 x^2 \cdot \left(\cancel{A e^{-\beta x^2}}\right) = E_0 \cdot \left(\cancel{A e^{-\beta x^2}}\right)$$

$$E_0 = \frac{\hbar\omega}{2} - \cancel{\frac{m\omega^2 x^2}{2}} + \cancel{\frac{1}{2}m\omega^2 x^2} \Rightarrow \boxed{E_0 = \frac{\hbar\omega}{2}}$$

EUF 2012 · 1Sem 25/03/2016

d) $V(x) = V_0 \exp\left(-\frac{x^2}{b^2}\right)$

$$E_0^{(1)} = \langle \psi_0 | V | \psi_0 \rangle = \int_{-\infty}^{\infty} A^2 \cdot V_0 e^{-\frac{x^2}{b^2}} \cdot e^{-\frac{m\omega x^2}{2\hbar}} dx$$

$$E_0^{(1)} = A^2 V_0 \int_{-\infty}^{\infty} e^{x^2\left(-\frac{1}{b^2} - \frac{m\omega}{2\hbar}\right)} dx \Rightarrow E_0^{(1)} = A^2 V_0 \cdot \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}} \qquad \alpha = \frac{2\hbar + m\omega b^2}{2\hbar b^2}$$

$\left(-\frac{1}{b^2} - \frac{m\omega}{2\hbar}\right) \to \frac{-2\hbar - m\omega b^2}{2\hbar b^2}$

$$E_0^{(1)} = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{2}} \cdot \left(\frac{\pi 2\hbar b^2}{2\hbar + m\omega b^2}\right)^{\frac{1}{2}} \cdot V_0 = \boxed{\left(\frac{2m\omega b^2}{2\hbar + m\omega b^2}\right)^{\frac{1}{2}} \cdot V_0}$$ deslocamento de energia.

Energia total

$$\boxed{E_0 = E_0^{(0)} + E_0^{(1)}}$$

Q5.

a) $\vec{\mu} = \gamma \vec{S}$

$$H = -\vec{\mu} \cdot \vec{B} = -\gamma \vec{B} \cdot \vec{S} \Rightarrow H_z = -\gamma \vec{B} \vec{S}_z = -\gamma \vec{B} \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

b) $\hat{H}|\psi\rangle = E_n |\psi\rangle$

$$-\gamma \vec{B} \frac{\hbar}{2} \begin{vmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{vmatrix} = 0 \Rightarrow \lambda = \mp \gamma \vec{B} \frac{\hbar}{2}$$

$$\boxed{E_n = \pm \gamma \vec{B} \frac{\hbar}{2}}$$

$\lambda_1 = -\gamma \vec{B} \frac{\hbar}{2}$

$$\begin{bmatrix} -\gamma \vec{B} \frac{\hbar}{2} & 0 \\ 0 & \gamma \vec{B} \frac{\hbar}{2} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = -\gamma \vec{B} \frac{\hbar}{2} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$-\gamma \vec{B} \frac{\hbar}{2} a = -\gamma \vec{B} \frac{\hbar}{2} a \Rightarrow |E_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\gamma \vec{B} \frac{\hbar}{2} b = -\gamma \vec{B} \frac{\hbar}{2} b \qquad |E_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

c) $\chi(0) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\alpha} \end{pmatrix} = \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ e^{i\alpha} \end{pmatrix}} = \frac{1}{\sqrt{2}}|E_1\rangle + \frac{e^{-i\alpha}}{\sqrt{2}}|E_2\rangle$

$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = H\Psi(x,t) \Rightarrow \chi(t) = e^{\frac{-iHt}{\hbar}} \cdot \chi(0)$

$$\boxed{\chi(t) = \frac{e^{\frac{i\gamma B t}{2}}}{\sqrt{2}}|E_1\rangle + \frac{e^{-i\alpha} \cdot e^{\frac{-i\gamma B t}{2}}}{\sqrt{2}}|E_2\rangle}$$ or

$i\hbar\begin{pmatrix} \dot{c}_1 \\ \dot{c}_2 \end{pmatrix} = \begin{pmatrix} +\frac{\gamma B\hbar}{2} & 0 \\ 0 & +\frac{\gamma B\hbar}{2} \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

$i\hbar \dot{c}_1 = -\frac{\gamma B\hbar}{2} c_1 \rightsquigarrow \frac{dc_1}{c_1} = \frac{i\gamma B}{2} dt \Rightarrow \boxed{c_1 = Ae^{\frac{i\gamma B}{2}t}}$

$i\hbar \dot{c}_2 = \frac{\gamma B\hbar}{2} c_2 \rightsquigarrow \frac{dc_2}{c_2} = -\frac{i\gamma B}{2} dt \Rightarrow \boxed{c_2 = Be^{-\frac{i\gamma B}{2}t}}$

$|\Psi(t)\rangle = Ae^{\frac{i\gamma Bt}{2}}|+a\rangle + Be^{\frac{-i\gamma Bt}{2}}|-a\rangle$

$|\Psi(0)\rangle = A|+a\rangle + B|-a\rangle \Rightarrow A = \frac{1}{\sqrt{2}}, \; B = \frac{e^{-i\alpha}}{\sqrt{2}}$ //

d) $S_x = \frac{1}{\sqrt{2}}|E_1\rangle + \frac{1}{\sqrt{2}}|E_2\rangle$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 1\begin{bmatrix} a \\ b \end{bmatrix} \Rightarrow b = a \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow |\hat{+}_x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|\langle S_x + \frac{\hbar}{2}|\Psi(t)\rangle|^2 \; \left|\frac{1}{2}\left[\langle E_1| + \langle E_2|\right] \cdot \left[e^{\frac{i\gamma Bt}{2}}|E_1\rangle + e^{-i(\frac{\gamma Bt}{2}+\alpha)}|E_2\rangle\right]\right.$

$= \left|\frac{1}{2}\left[e^{\frac{i\gamma Bt}{2}} + e^{-i(\frac{\gamma Bt}{2}+\alpha)}\right]\right|^2 = \frac{1}{4}\left[e^{\frac{i\gamma Bt}{2}} + e^{\frac{-i\gamma Bt}{2}} \cdot e^{-i\alpha}\right]\left[e^{\frac{-i\gamma Bt}{2}} + e^{\frac{i\gamma Bt}{2}} \cdot e^{i\alpha}\right]$

ECF 2012 - 1sem 25/03

$\rho_+(t) = \frac{1}{4}\left(1 + 1 + e^{i\gamma\beta t}\cdot e^{i\alpha} + e^{-i\gamma\beta t}\cdot e^{-i\alpha}\right)$

$\phi_1 = \frac{i\gamma\beta t}{2}$ $\phi_2 = -\frac{i\beta t \gamma}{2} - i\alpha$ $\cos^2 x = \frac{1+\cos 2x}{2}$

$= \frac{1}{4}\left[2 + \left(e^{i(\alpha+\gamma\beta t)} + e^{-i(\alpha+\gamma\beta t)}\right)\right]$ $2\cdot\cos(\alpha+\gamma\beta t) = 2\cos^2 x = 1+\cos 2x$

$= \frac{1}{2}\left[1 + \cos(\gamma\beta t + \alpha)\right] = \frac{1}{2}\cdot\left[2\cdot\cos^2\left(\frac{\gamma\beta t+\alpha}{2}\right)\right]$

$\boxed{= \cos^2\left(\frac{\gamma\beta t+\alpha}{2}\right)}$

Q10.

$P_0 = \frac{nRT_i}{V_0}$ $P_f = \frac{2nRT}{V_0}$ $P_0V_0 = P_f V_f$ $P_f = 2P_0$

a) $dU = \cancel{dQ} + dW$

$\Delta U = -W \rightarrow \Delta U = -p\,dV$

$\Delta U = \int_{V_0}^{V_0/2} \frac{dV\, nRT}{V} = nRT \ln\left(\frac{1}{2}\right)$

b) $Q = ST$

$S = \frac{Q}{T} \Rightarrow \Delta S = 0$

V_0 | ter